Brane-induced Skyrmions

– Baryons in Holographic QCD –

Kanabu Nawa, Hideo Suganuma and Toru Kojo

Department of Physics, Kyoto University, Kyoto 606-8502, Japan

We study baryons in holographic QCD with $D4/D8/\overline{D8}$ multi D brane system. In holographic QCD, the baryon appears as a topologically non-trivial chiral soliton in a four-dimensional effective theory of mesons, which is called 'Brane-induced Skyrmion'. We derive and calculate the Euler-Lagrange equation for the hedgehog configuration with chiral profile $F(r)$ and ρ-meson profile $G(r)$, and obtain the soliton solution of the holographic QCD.

§1. Holographic QCD, color confinement and chiral symmetry breaking

Based on the recent remarkable progress in the concept of gauge/gravity duality, Sakai-Sugimoto succeeded to construct the low-energy theory of massless QCD from the multi D-brane system consisting of $D4/D8/\overline{D8}$ in type IIA superstring theory.[1)]

In the $D4/D8/\overline{D8}$ holographic QCD, $D4/D8/\overline{D8}$-branes are placed as Table I, and $D4$ branes are compactified along an extra direction $x_4 \equiv \tau$ with the Kaluza-Klein mass scale as $\tau \sim \tau + 2\pi M_{\rm KK}^{-1}$, to take away the extra-modes for massless QCD. The compositions of the four-dimensional massless QCD, *i.e.*, gluons and massless quarks, are represented as the fluctuation modes of open strings on N_c-folded $D4$ branes and N_f-folded $D8$ and $\overline{D8}$ branes as Fig.1. In the $D4/D8/\overline{D8}$ holographic QCD with large N_c, $D4$ branes are represented by the classical supergravity background with the concept of 'gauge/gravity duality', and back reaction from the probe $D8$ and $\overline{D8}$ branes to the total system can be neglected as a probe approximation.

	0	1	2	3	4	5	6	7	8	9
$D4$	○	○	○	○	○					
$D8$-$\overline{D8}$	○	○	○	○		○	○	○	○	○

Table I. The space-time extension of the $D4$ brane and $D8$-$\overline{D8}$ branes to construct massless QCD. The circle denotes the extended direction of each D brane. $x_{0\sim 3}$ correspond to flat space-time.

In the presence of N_c-folded $D4$ brane, the radial direction U in extra-dimensions $x_{5\sim 9}$ is bounded from below like $U \geq U_{\rm KK}$ as a 'horizon' in ten-dimensional space-time. Then, probe $D8$ and $\overline{D8}$ branes are interpolated with each other in this curved space-time as in Fig. 2. This interpolation indicates symmetry breaking of $\mathrm{U}(N_f)_{D8} \times \mathrm{U}(N_f)_{\overline{D8}}$ into $\mathrm{U}(N_f)_{D8}$, which can be regarded as the holographic manifestation of spontaneous chiral symmetry breaking.

Furthermore, color confinement is realized in the holographic QCD as follows. Since color quantum number is carried only by N_c-folded $D4$ branes, colored objects appear as the fluctuation modes of open strings with at least one end located on N_c-folded $D4$ branes, *e.g.*, gluons from 4-4 strings and quarks from 4-8 strings. Therefore, in the supergravity background of $D4$ brane, these colored objects appearing around $D4$ branes would locate behind the horizon $U_{\rm KK}$ and become invisible from outside, which can be interpreted as color confinement at low-energy scale.

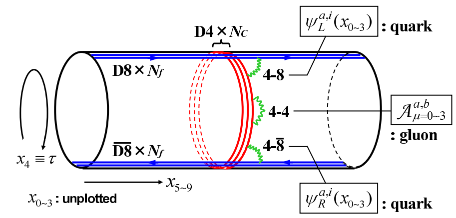

Fig. 1. Multi D brane configurations of the $D4/D8/\overline{D8}$ holographic QCD. N_c-folded $D4$ branes and N_f-folded $D8$-$\overline{D8}$ branes. Gluons and quarks appear as 4-4, 4-8 and 4-$\overline{8}$ strings shown by waving lines.

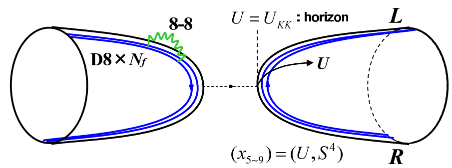

Fig. 2. Probe $D8$ branes with $D4$ supergravity background. The radial coordinate U in the extra five dimensions $x_{5\sim9}$ is bounded from below by a horizon $U_{\rm KK}$ as $U \geq U_{\rm KK}$. Color-singlet mesons appear from 8-8 strings shown by the waving line.

From these considerations, one may see that chiral symmetry breaking and color confinement occur simultaneously; two independent chirality spaces are connected by the 'worm hole' into which colored objects are absorbed as shown in Fig.2.

After the supergravity description of background $D4$ branes, there only appear color-less objects as the fluctuation modes of residual probe $D8$ branes: mesons and also baryons. Since large-N_c QCD becomes equivalent with the weak-coupling system of mesons and glueballs,[2)] baryons do not directly appear as the dynamical degrees of freedom but appear as some soliton-like topological objects[3)] in this large-N_c holographic QCD. In this paper, we study the baryon as a topologically non-trivial chiral soliton, which is called as 'Brane-induced Skyrmion', in the four-dimensional meson effective theory induced by the $D4/D8/\overline{D8}$ holographic QCD.[4)]

The existence of $D4$ supergravity background is reflected on the metric of the non-abelian Dirac-Born-Infeld (DBI) action of $D8$ brane in nine-dimensional space-time. After integrating out extra four-dimensional angular coordinates around $x_{5\sim9}$, the effective action of $D8$ brane is reduced into the five-dimensional Yang-Mills theory with flat four-dimensional space-time $(t, \mathbf{x})$ and other extra fifth dimension z with curved measure as[1)]

$$S_{D8}^{\rm DBI} = \kappa \int d^4x dz {\rm tr} \left\{ \frac{1}{2} K(z)^{-\frac{1}{3}} F_{\mu\nu} F_{\mu\nu} + K(z) F_{\mu z} F_{\mu z} \right\} + O(F^4), \qquad (1{\cdot}1)$$

where A_M-independent part is abbreviated. $K(z) \equiv 1+z^2$ is the non-trivial curvature in the fifth direction z induced by the supergravity background of $D4$ branes, and $\kappa \equiv \frac{\lambda N_c}{108\pi^3}$ with the 't Hooft coupling $\lambda \equiv g^2_{\rm YM} N_c$. We here take $M_{\rm KK} = 1$ unit. We now treat non-trivial leading order $O(F^2)$ of the DBI action above, corresponding to the leading order of $1/N_c$ and $1/\lambda$ expansions in the holographic model, for the argument of non-perturbative (strong coupling) properties of QCD.

To obtain the four-dimensional effective theory with definite parity and G-parity of QCD, we perform the mode expansion of the five-dimensional gauge field $A_M(x_N)$ $(M, N = 0 \sim 4)$ with respect to the extra-coordinate z by using proper parify-definite orthogonal basis $\psi_\pm(z)$ and $\psi_n(z)$ $(n = 1, 2, \cdots)$ as

$$A_\mu(x_N) = l_\mu(x_\nu)\psi_+(z) + r_\mu(x_\nu)\psi_-(z) + \sum_{n\geq1} B_\mu^{(n)}(x_\nu)\psi_n(z), \qquad (1{\cdot}2)$$

$$l_\mu(x_\nu) \equiv \frac{1}{i}\xi^{-1}(x_\nu)\partial_\mu\xi(x_\nu),\ \ r_\mu(x_\nu) \equiv \frac{1}{i}\xi(x_\nu)\partial_\mu\xi^{-1}(x_\nu),\ \ \xi(x_\nu) \equiv e^{i\pi(x_\nu)/f_\pi}, \quad (1\cdot 3)$$

where '$A_z = 0$ and $\xi_+^{-1} = \xi_-$' gauge is taken.[1),4)] To diagonalize the five-dimensional Yang-Mills theory with curved measure $K^{-\frac{1}{3}}(z)$ and $K(z)$ in the fifth direction z, the basis $\psi_n(z)$ are taken to be the normalizable eigen-function satisfying

$$-K(z)^{\frac{1}{3}}\frac{d}{dz}\left\{K(z)\frac{d\psi_n}{dz}\right\} = \lambda_n\psi_n, \qquad (0 < \lambda_1 < \lambda_2 < \cdots) \qquad (1\cdot 4)$$

and the basis $\psi_\pm(z) \equiv \frac{1}{2} \pm \frac{1}{\pi}\arctan z$ are taken as zero modes of this eigen-equation.

In the holographic QCD, $B_\mu^{(n=1,2,\cdots)}$ are regarded as the infinite excitation modes of (axial-)vector mesons. Here, the basis $\psi_\pm$ and ψ_n can be regarded as the 'wave-function' of pions and (axial-)vector mesons in the fifth dimension. The mass of (axial-)vector mesons is given by the eigen-value of the basis ψ_n in Eq.(1·4) as $m_n = \lambda_n$ in this holographic QCD, which indicates that a large oscillation in the extra fifth direction induces a large mass in the four-dimensional theory. As for pions, slightly oscillating component $\psi_\pm$ of pion wave functions are, in fact, the zero mode of Eq.(1·4) with curved measure, so that pions appear as massless objects, corresponding to the 'geodesic' of curved five-dimensional space-time.[4)]

The smaller overlapping of wave functions in the extra fifth direction between pions and 'largely oscillating' heavier (axial-)vector mesons predicts the smaller coupling constant between them[4),5)] through the projection of the action into flat four-dimensional space-time. Therefore, for the study of chiral solitons, which consist of large-amplitude pion fields, we can expect smaller effects from heavier (axial-)vector mesons, so that it would be enough to consider only pions and ρ mesons as the lowest massive mode $B_\mu^{(1)}$ ($\equiv \rho_\mu$) in the holographic QCD.[4)]

§2. Brane-induced Skyrmion and its properties

Now, substituting the mode expansion of the gauge fields (1·2) in the five-dimensional Yang-Mills theory, we get the four-dimensional effective Lagrangian with pions and ρ mesons from holographic QCD, without small amplitude expansion, as

$$\begin{aligned}
&\mathcal{L}_{D8}^{\rm DBI} = \kappa\int dz{\rm tr}\left\{\frac{1}{2}K(z)^{-\frac{1}{3}}F_{\mu\nu}F_{\mu\nu} + K(z)F_{\mu z}F_{\mu z}\right\}\\
&= \frac{f_\pi^2}{4}{\rm tr}\left(L_\mu L_\mu\right) - \frac{1}{32e^2}{\rm tr}\left[L_\mu, L_\nu\right]^2 + \frac{1}{2}{\rm tr}\left(\partial_\mu\rho_\nu - \partial_\nu\rho_\mu\right)^2 + m_\rho^2{\rm tr}\left(\rho_\mu\rho_\mu\right)\\
&+ i\,g_{3\rho}{\rm tr}\left\{\left(\partial_\mu\rho_\nu - \partial_\nu\rho_\mu\right)\left[\rho_\mu, \rho_\nu\right]\right\} - \frac{1}{2}g_{4\rho}{\rm tr}\left[\rho_\mu, \rho_\nu\right]^2 - ig_1{\rm tr}\left\{\left[\alpha_\mu, \alpha_\nu\right]\left(\partial_\mu\rho_\nu - \partial_\nu\rho_\mu\right)\right\}\\
&+\ g_2{\rm tr}\left\{\left[\alpha_\mu, \alpha_\nu\right]\left[\rho_\mu, \rho_\nu\right]\right\} + 2g_3{\rm tr}\left\{\left[\alpha_\mu, \alpha_\nu\right]\left[\beta_\mu, \rho_\nu\right]\right\} + i2g_4{\rm tr}\left\{\left(\partial_\mu\rho_\nu - \partial_\nu\rho_\mu\right)\left[\beta_\mu, \rho_\nu\right]\right\}\\
&- 2g_5{\rm tr}\left\{\left[\rho_\mu, \rho_\nu\right]\left[\beta_\mu, \rho_\nu\right]\right\} - \frac{g_6}{2}{\rm tr}\left(\left[\alpha_\mu, \rho_\nu\right] + \left[\rho_\mu, \alpha_\nu\right]\right)^2 - \frac{g_7}{2}{\rm tr}\left(\left[\beta_\mu, \rho_\nu\right] + \left[\rho_\mu, \beta_\nu\right]\right)^2 \ (2\cdot 1)
\end{aligned}$$

with $\alpha_\mu(x_\mu) \equiv l_\mu(x_\mu) - r_\mu(x_\mu)$, $\beta_\mu(x_\mu) \equiv \frac{1}{2}\left\{l_\mu(x_\mu) + r_\mu(x_\mu)\right\}$ and $L_\mu \equiv \xi^{-1}\alpha_\mu\xi$. As a remarkable fact, constants f_π, m_ρ, e, $g_{3\rho}$, $g_{4\rho}$ and $g_{1\sim 7}$ can be written by the basis $\psi_\pm$ and ψ_1, *e.g.*, $g_{3\rho} \equiv \kappa \int dz K(z)^{-\frac{1}{3}}\psi_1^3$. The holographic QCD has, in fact, just two parameters κ and $M_{\rm KK}$, and therefore all the coupling constants in the action (2·1) are uniquely determined, by fixing two parameters like experimental inputs for f_π and m_ρ. Note also that, because of $g_{3\rho}^2 \neq g_{4\rho}$, the ρ-meson part in the four-dimensional effective theory differs from the massive Yang-Mills theory.

For the static soliton solution of the action (2·1), we take the hedgehog configuration Ansatz for pion field $U(\mathbf{x})$ and ρ meson field $\rho_\mu(\mathbf{x})$,

$$U^\star(\mathbf{x}) = e^{i\tau_a \hat{x}_a F(r)}, \quad \rho_0^\star(\mathbf{x}) = 0, \quad \rho_i^\star(\mathbf{x}) = \rho_{ia}^\star(\mathbf{x})\tau_a = \left\{\varepsilon_{iab}\hat{x}_b \tilde{G}(r)\right\}\tau_a, \tag{2·2}$$

where $F(r)$ $(r \equiv |\mathbf{x}|)$ is a dimension-less function with boundary conditions $F(0) = \pi$ and $F(\infty) = 0$, giving topological charge equal to unity as a unit baryon number.

We derive the energy density $\varepsilon[F(r), \tilde{G}(r)]$ of the Brane-induced Skyrmion as

$$\begin{aligned}
r^2\varepsilon[F(r), \tilde{G}(r)] &= \frac{f_\pi^2}{4}\left[2\left(r^2F'^2 + 2\sin^2 F\right)\right] + \frac{1}{32e^2}\left[16\sin^2 F\left(2F'^2 + \frac{\sin^2 F}{r^2}\right)\right] \\
&+ \frac{1}{2}\left[8\left\{3\tilde{G}^2 + 2r\tilde{G}(\tilde{G}') + r^2\tilde{G}'^2\right\}\right] + m_\rho^2\left[4r^2\tilde{G}^2\right] - g_{3\rho}\left[16r\tilde{G}^3\right] + \frac{1}{2}g_{4\rho}\left[16r^2\tilde{G}^4\right] \\
&+ g_1\left[16\left\{F'\sin F\cdot\left(\tilde{G} + r\tilde{G}'\right) + \sin^2 F\cdot\tilde{G}/r\right\}\right] - g_2\left[16\sin^2 F\cdot\tilde{G}^2\right] \\
&- g_3\left[16\sin^2 F\cdot(1-\cos F)\,\tilde{G}/r\right] - g_4\left[16\,(1-\cos F)\,\tilde{G}^2\right] + g_5\left[16r\,(1-\cos F)\,\tilde{G}^3\right] \\
&+ g_6\left[16r^2F'^2\tilde{G}^2\right] + g_7\left[8\,(1-\cos F)^2\,\tilde{G}^2\right].
\end{aligned} \tag{2·3}$$

We note that Brane-induced Skyrmion has a scaling property.[4] By rewriting energy and length in 'Adkins-Nappi-Witten (ANW) unit' [6] as $E_{\rm ANW} \equiv \frac{f_\pi}{2e}(\propto \kappa M_{\rm KK})$ and $r_{\rm ANW} \equiv \frac{1}{ef_\pi}(\propto \frac{1}{M_{\rm KK}})$, all the physical parameters like f_π, m_ρ, e (and also $m_{a_1}, m_{\rho'}, m_{a_1'}, \cdots$) are uniquely determined by the unit of $E_{\rm ANW}$ and $r_{\rm ANW}$, because the holographic QCD has just two parameters, κ and $M_{\rm KK}$. This scaling property of Brane-induced Skyrmion is a remarkable consequence of holographic framework.

With the hedgehog configuration, we study baryons as Brane-induced Skyrmions in the large-N_c holographic QCD, and obtain the stable soliton solution with chiral profile $F(r)$ and rescaled ρ-meson profile $\widehat{G}(r) \equiv \frac{1}{\sqrt{\kappa}}\tilde{G}(r)$, as shown in Fig.3. We

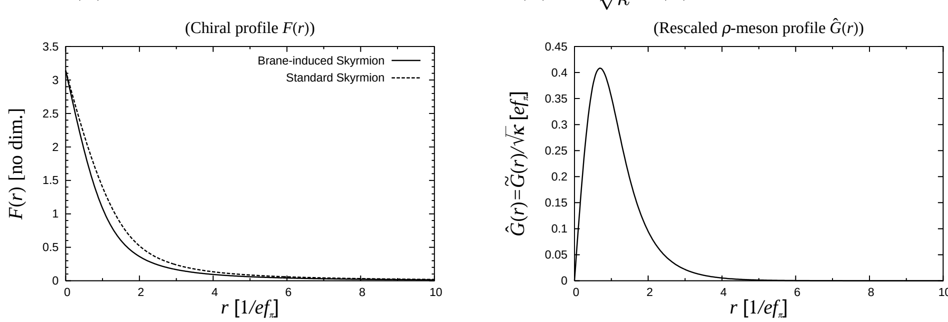

Fig. 3. The chiral profile $F(r)$ and rescaled ρ-meson profile $\widehat{G}(r)$ of Brane-induced Skyrmion as the hedgehog soliton solution in the holographic QCD. The dashed curve in the left figure denotes the chiral profile of standard Skyrmion without ρ mesons.

numerically obtain the total energy (mass) of Brane-induced Skyrmion in ANW unit as $E \simeq 1.115 \times 12\pi^2 \left[\frac{f_\pi}{2e}\right]$ (c.f. $E \simeq 1.231 \times 12\pi^2 \left[\frac{f_\pi}{2e}\right]$ for standard Skyrmion[6)]). The mass of Brane-induced Skyrmion is reduced by $\sim 10\%$ relative to the standard Skyrmion because of the interactions between pions and ρ mesons in the meson effective action (2·1) or the energy density (2·3) induced by the holographic QCD.

Figure 4 shows the energy density of Brane-induced Skyrmion. From the energy-density distribution, we estimate the radius of the Brane-induced Skyrmion in ANW unit as $\sqrt{\langle r^2 \rangle} \simeq 1.268 \left[\frac{1}{ef_\pi}\right]$ (c.f. $\sqrt{\langle r^2 \rangle} \simeq 1.422 \left[\frac{1}{ef_\pi}\right]$ for standard Skyrmion). In the Brane-induced Skyrmion, some part of total mass is carried by the heavy ρ-meson in the soliton core, which gives the shrinkage of the total size by $\sim 10\%$ relative to the standard Skyrmion. We show ρ-meson contributions to the energy density (2·3) in the Brane-induced Skyrmion in Fig.5, which indicates that ρ-meson components are rather active in the core region of baryons through various interaction terms in the action (2·1). This active ρ-meson component inside baryons may be a new striking picture for baryons suggested from the holographic QCD.

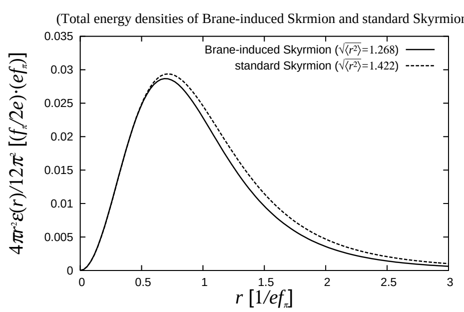

Fig. 4. The energy density profile $4\pi r^2 \varepsilon(r)$ (per BPS value of $12\pi^2$) of Brane-induced Skyrmion with that of standard Skyrmion.

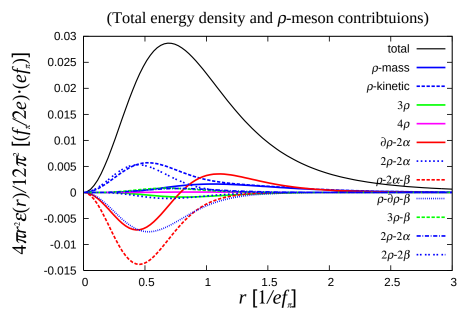

Fig. 5. Contributions of ρ-meson interaction terms in (2·1) to the energy density of Brane-induced Skyrmion with total energy density.

By taking the experimental inputs as $f_\pi = 92.4$MeV and $m_\rho = 776$MeV, all the variables in the holographic QCD are uniquely determined as $\kappa \simeq 7.46 \times 10^{-3}$, $M_{\rm KK} \simeq 948$MeV, $e \simeq 7.315$. With these variables, we find reasonable mass $M_{\rm HH} \simeq 834$MeV and small radius $\sqrt{\langle r^2 \rangle} \simeq 0.37$fm for the hedgehog Brane-induced Skyrmion.

To summarize, we have studied Brane-induced Skyrmions, *i.e.*, baryons in the holographic QCD with $D4/D8/\overline{D8}$ multi D brane system, and we have numerically obtained the hedgehog soliton solution of the holographic QCD.

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References

1) T. Sakai and S. Sugimoto, Prog. Theor. Phys. **113**, 843 (2005); *ibid.*, **114**, 1083 (2006).
2) G. 't Hooft, Nucl. Phys. B **72**, 461 (1974); B **75**, 461 (1974).
3) T.H.R.Skyrme, Nucl. Phys. **31**, 556 (1962); K.F.Liu (ed.), 'Chiral Solitons' (WSPC, 1987).
4) K. Nawa, H. Suganuma and T. Kojo, 'Baryons in holographic QCD', hep-th/0612187.
5) D. T. Son and M. A. Stephanov, Phys. Rev. D **69**, 065020 (2004).
6) G. S. Adkins, C. R. Nappi and E. Witten, Nucl. Phys. B **228**, 552 (1983).